Development of a Kinematic Model based on Bézier Curves for Improvement of Safe Trajectories in Active Orthosis Walking Tasks

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Abstract— **This work presents a kinematic walking model for an active orthosis with 4 degrees of freedom based on Bézier curves as foot trajectory. Moreover, the proposed model reinforces the importance of this model for crossing holes and other obstacles. Gravitational reactions and balance control are not considered in this paper, because the user is supported by a couple of crutches. The proposed method was simulated based on Ortholeg orthosis parameters with 20kg of structural weight, for users from 1,55m to 1,70m height and weight up to 65kg. Simulation experiments shown that for walking task, including crossing holes and small obstacles, the proposed model obtained good results.**

Keywords: kinematics walking model; Bézier curves trajectory; orthosis modelling; assistive robotics.

I. INTRODUCTION

In recent years, the development of assistive devices is growing in the academic community. Besides of other robotic systems, the assistive technology that includes robotics systems must be based mainly on reliability, robustness and safety just as several devices around the world that tries to fill this field such as: smart wheelchair [1], active orthosis using biosignals [2] and mechanical/robotic prothesis for arms, hands and foot [3].

There are several kinematic walking models to active orthosis or humanoid robots. Uchiyama et al., simulated a walking motion for a powered orthosis using a couple of central pattern generators (CPG) [4]. Haghighi and Nekoui, used Cubic Polynomial method as a foot trajectory generator for one humanoid robot eight joints [5]. Santos et al., proposed a new architecture for a biped robot with seven DOF per each leg and one DOF corresponding to the toe joint, dividing the walking gait into the Sagittal and Frontal planes [6]. Marques et al., presented a different method to model kinematics of humanoid robots avoiding the restriction to the frontal and sagittal planes [7]. Rameez and Khan, presented dynamic equations of motion and its Matlab simulation of joints position using equations with forward kinematics and inverse kinematics [8].

With the Bézier curves other approaches may define a geometric representation of trajectories and curves such as: Cubic Polynomial method [5], Nelson polynomials [9] and polynomial spirals [10].

This work presents a solution for foot trajectory modeling based on Bézier curves and inverse kinematics, applied to a 4 degrees of freedom active orthosis on walking task, crossing holes and small obstacles. The gravitational reactions and balance control are not considered, since the user walking is supported by a couple of crutches.

II. THE ORTHOLEG PROTOTYPE

The proposed inverse kinematic model is based on an active orthosis parameters with 4 degrees of freedom (DoF) named Ortholeg v1.0. It has four actuators placed in the knees and hips controlled on sagittal plane, 20kg of structural weight and being applied to users within a height range between 1,55m to 1,70m and 65kg weight. It is able to perform movements such as straight walk, sit and stand up [11].

Furthermore, other Ortholeg version is being developed by the group [12] as shown in Figure 1.

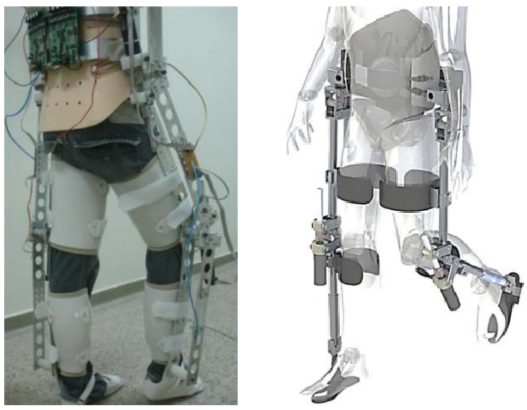

Fig. 1. Orthesis Ortholeg v1.0 (left) and v2.0 (right).

III. BÉZIER CURVES AS FOOT TRAJECTORY

Bézier curves are a powerful tool for constructing free-form curves and surface. It have a fundamental importance for computer aided geometric design (CAGD) and computer graphics (CG) [13]. In this work are used to define the foot trajectories of an active orthosis, giving support to walking and crossing obstacles tasks.

Mathematically the Bézier curves are based on the *binomial coefficients* and are defined by a set of *control points* $\{\mathcal{P}_0, \mathcal{P}_1, \mathcal{P}_2, .., \mathcal{P}_n\}$ where n represents the order's curve. For

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each n should apply $n+1$ *control points* (i.e. $count(\mathcal{P}) = n+1$) for each curve defined by Equation 1.

$$\mathcal{B}(t,n) = \sum_{i=0}^{n} \binom{n}{i} (1-t)^{n-i} t^i \mathcal{P}_i, 0 \leq t \leq 1 \tag{1}$$

where t represents a parametrization value along the curve $\mathcal{B}(t,n)$ followed by orthosis foot, applied for both axis x and y as defined by,

$$\mathcal{B}_X(t,n) = \sum_{i=0}^{n} \binom{n}{i} (1-t)^{n-i} t^i \mathcal{P}_{Xi} \tag{2}$$

$$\mathcal{B}_Y(t,n) = \sum_{i=0}^{n} \binom{n}{i} (1-t)^{n-i} t^i \mathcal{P}_{Yi} \tag{3}$$

where $\mathcal{P}_{Xi}$ and $\mathcal{P}_{Yi}$ represents the *control points* vectors in the axis x and y respectively.

The matrix $\mathcal{B}(t,n)$ represented by $[\mathcal{B}_X, \mathcal{B}_Y]^T$, is processed by the orthosis control for each leg step routine, varying its values according to the user needs, that selects the step distance.

IV. KINEMATIC WALKING MODEL

The proposed new approach of kinematic model to design the foot trajectory of an active orthosis, is based on Bézier curves concepts. It have low complexity of execution being useful to orthosis walking task. The mathematical modeling is have two phases for one step cycle by time:

- Mathematical modeling from **moving leg** (Leg_M), i.e. the leg that moves along the trajectory $\mathcal{B}(t,n)$;
- Mathematical modeling from **support leg** (Leg_S), i.e. the leg that keeps fixed while other leg reach the last *control point* $\mathcal{P}_n$ inside the $\mathcal{B}(t,n)$.

When a new step cycle starts, both legs change its roles, i.e. $Leg_M \rightarrow Leg_S$ and $Leg_S \rightarrow Leg_M$.

Figure 2 shows all joints angles such as, α_1, α_2 and α_3 from support leg, and θ_1, θ_2, θ_3, from moving leg.

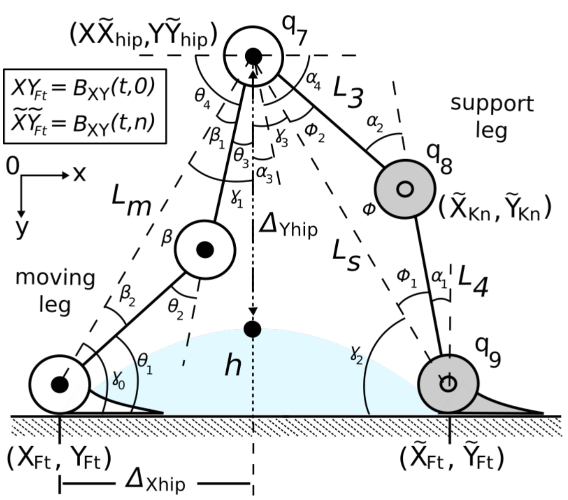

Fig. 2. Biped system used in the mathematical modeling for one step cycle in sagittal plane.

The initial points $\mathcal{P}_{X0} = \mathcal{B}_X(t,0)$ and $\mathcal{P}_{Y0} = \mathcal{B}_Y(t,0)$, represent X_{Ft} and Y_{Ft} respectively; and $\tilde{X}_{Ft}$ and $\tilde{Y}_{Ft}$ represent the final step points $\mathcal{P}_{Xn} = \mathcal{B}_X(t,n)$ and $\mathcal{P}_{Yn} = \mathcal{B}_Y(t,n)$ respectively.

A. Mathematical Modeling for Leg_M

The Leg_M parameters such as angles and coordinates are defined as below.

$$L_T = L_1 + L_2 \tag{4}$$

$$\Delta_{Xhip} = X_{hip} - X_{Ft} \tag{5}$$

$$\Delta_{Yhip} = L_T - Y_{Ft} \tag{6}$$

$$L_m = \sqrt{{\Delta_{Xhip}}^2 + {\Delta_{Yhip}}^2} \tag{7}$$

Using the law of cosines, the internal angles β, β_1 and β_2, are defined by Equations 8-10.

$$\beta = \sqrt{\frac{\cos^{-1}(-{L_m}^2 + {L_2}^2 + {L_1}^2)}{2L_1L_2}} \tag{8}$$

$$\beta_1 = \sqrt{\frac{\cos^{-1}({L_m}^2 + {L_1}^2 - {L_2}^2)}{2L_mL_1}} \tag{9}$$

$$\beta_2 = \sqrt{\frac{\cos^{-1}({L_m}^2 + {L_2}^2 - {L_1}^2)}{2L_mL_2}} \tag{10}$$

$$\gamma_0 = \tan^{-1}(\frac{\Delta_{Yhip}}{\Delta_{Xhip}}) \tag{11}$$

$$\gamma_1 = \tan^{-1}(\frac{\Delta_{Xhip}}{\Delta_{Yhip}}) \tag{12}$$

$$\theta_1 = \gamma_0 - \beta_2 \tag{13}$$

$$\theta_2 = \pi - \beta \tag{14}$$

$$\theta_3 = \gamma_1 - \beta_1 \tag{15}$$

$$\theta_4 = \frac{\pi}{2} - \theta_3 \tag{16}$$

The intersection of straight lines L_1 and L_2, defines the coordinates for the knee from moving leg, as shown below.

$$X_{Kn} = \frac{\tan\theta_4 X_{hip} - \tan\theta_1 \mathcal{B}_X(t,n) - \Delta_{Yhip} + \mathcal{B}_Y(t,n)}{\tan\theta_4 - \tan\theta_1} \tag{17}$$

$$Y_{Kn} = \tan\theta_1(X_{Kn} - \mathcal{B}_X(t,n)) + \mathcal{B}_Y(t,n) \tag{18}$$

B. Mathematical Modeling for Leg_S

The Leg_S parameters such as, angles and coordinates are defined as below, where $X_{Ft} = \mathcal{B}_X(t, 0)$.

$$\tilde{L}_T = L_T = L_1 + L_2 = L_3 + L_4 \tag{19}$$

$$\tilde{\Delta}_{Xhip} = X_{Ft} - \mathcal{B}_X(t, \frac{n}{2}) \tag{20}$$

$$\tilde{\Delta}_{Yhip} = \tilde{L}_T \tag{21}$$

$$L_s = \sqrt{\tilde{\Delta}^2_{Xhip} + \tilde{\Delta}^2_{Yhip}} \tag{22}$$

$$\phi = \sqrt{\frac{\cos^{-1}(-{L_s}^2 + {L_3}^2 + {L_4}^2)}{2L_3L_4}} \tag{23}$$

$$\phi_1 = \sqrt{\frac{\cos^{-1}({L_s}^2 + {L_3}^2 - {L_4}^2)}{2L_sL_3}} \tag{24}$$

$$\phi_2 = \sqrt{\frac{\cos^{-1}({L_s}^2 - {L_3}^2 + {L_4}^2)}{2L_sL_4}} \tag{25}$$

$$\gamma_2 = \tan^{-1}(\frac{\tilde{\Delta}_{Yhip}}{\tilde{\Delta}_{Xhip}}) \tag{26}$$

$$\gamma_3 = \tan^{-1}(\frac{\tilde{\Delta}_{Xhip}}{\tilde{\Delta}_{Yhip}}) \tag{27}$$

$$\alpha_1 = \frac{\pi}{2} - \gamma_2 - \phi_1 \tag{28}$$

$$\alpha_2 = \pi - \phi \tag{29}$$

$$\alpha_3 = \gamma_3 - \phi_2 \tag{30}$$

$$\alpha_4 = \frac{\pi}{2} - \alpha_3 \tag{31}$$

The intersection of the straight lines L_3 and L_4, defines the coordinates for the knee from support leg, as shown below, where $Y_{Ft} = \mathcal{B}_Y(t, 0)$.

$$\tilde{X}_{Kn} = \frac{\tan\alpha_4 \mathcal{B}_X(t, \frac{n}{2}) - \tan\alpha_1 X_{Ft} - \tilde{\Delta}_{Yhip} + Y_{Ft}}{\tan\alpha 4 - \tan\alpha_1} \tag{32}$$

$$\tilde{Y}_{Kn} = \tan\alpha_1(\tilde{X}_{Kn} - X_{Ft}) + Y_{Ft} \tag{33}$$

C. Model Particularities

This model produces a swing on the hips ($X\tilde{X}_{hip}$ and $Y\tilde{Y}_{hip}$ points) of the active orthosis (Equations 34-36), considering that the user is able to use crutches to ensure the balance and safety while walking or crossing obstacles.

$$X_{hip} = \tilde{X}_{hip} = \mathcal{B}_X(t, \frac{n}{2}) \tag{34}$$

$$Y_{hip} = \Delta_{Yhip} \tag{35}$$

$$\tilde{Y}_{hip} = \tilde{L}_T \tag{36}$$

The value of the central joint from the active orthosis model is represented by X_{ctr} and Y_{ctr}, defined below.

$$X_{ctr} = X_{hip} \tag{37}$$

$$Y_{ctr} = \frac{\Delta_{Yhip} + \tilde{\Delta}_{Yhip}}{2} \tag{38}$$

Figure 3 shows schematically the proposed model in execution, crossing obstacle (e.g. holes).

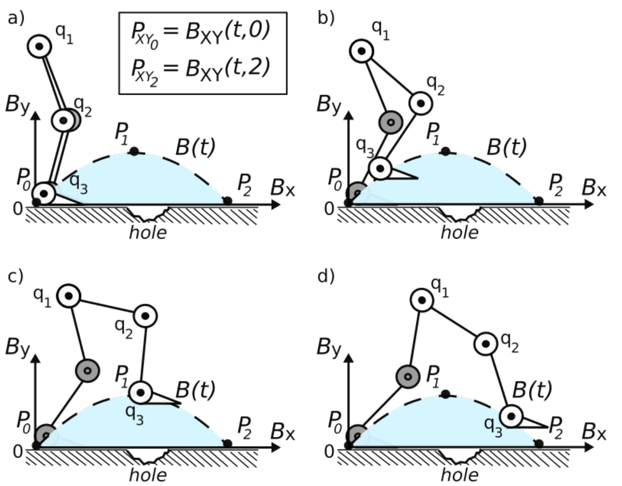

Fig. 3. Application of Bézier curves with three *control points* ($\mathcal{P}_0$, $\mathcal{P}_1$ and $\mathcal{P}_2$) in the orthosis walking and crossing hole tasks. a)The orthosis control starts one step; and d) end of one step while crossing a small hole.

V. RESULTS

The present mathematical modeling was applied to a 4 degrees of freedom active orthosis in walking tasks, including crossing holes and small obstacles as shown in Fig. 4, representing the execution of three Bézier curves and the trajectories of the knees (X_{Kn}, Y_{Kn}, $\tilde{X}_{Kn}$ and $\tilde{Y}_{Kn}$, from Leg_M and Leg_S respectively) and hips (X_{hip}, Y_{hip}, $\tilde{X}_{hip}$ and $\tilde{Y}_{hip}$, from Leg_M and Leg_S respectively).

Figures 5-6 show the joints angles θ_1, θ_2, θ_3 and θ_4 from Leg_M, and α_1, α_2, α_3 and α_4 from Leg_S, referent to three walking cycles in sagittal plane, where α_2 maintain constant because this joint was agreed to be fixed during all simulations, as presented in Ortholeg v1.0 structure.

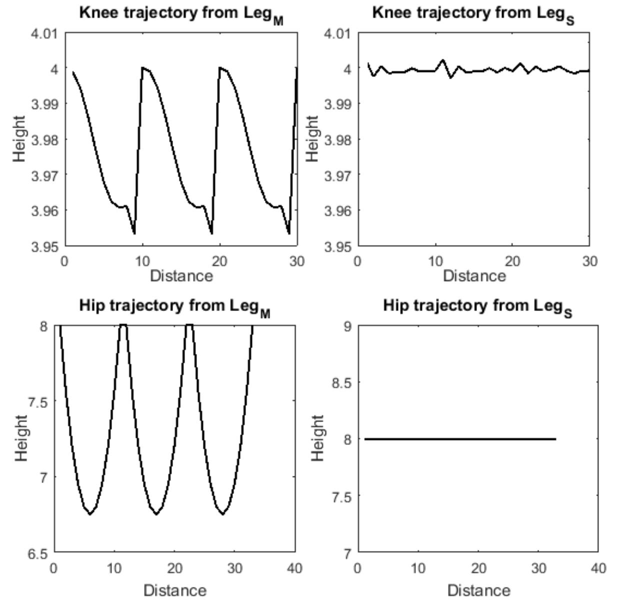

Fig. 4. Trajectory of the knees from Leg_M (up-left) and Leg_S (up-right), the hips from Leg_M (bottom-left) and Leg_S (bottom-right) from three walking cycles.

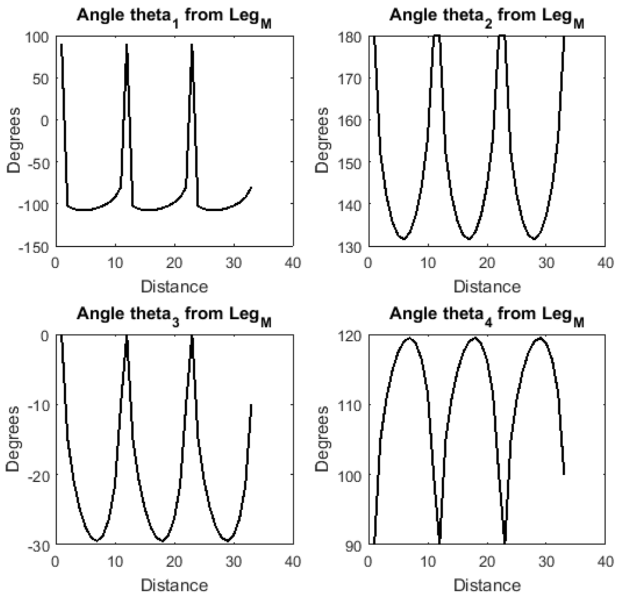

Fig. 5. Joints angles θ_1, θ_2, θ_3 and θ_4 from Leg_M from three walking cycles in sagittal plane.

VI. Conclusions and Future Works

This work presented a kinematic model for walking task based on Bézier curves to determine the foot trajectory of an active orthosis. The simulations achieved good results on walking and crossing obstacles (e.g. holes) tasks, according to step length defined by the user.

In future works, we intend to apply the same model in order to be used in a real orthosis like Ortholeg v1.0 and 2.0 to climb obstacles such as stairs.

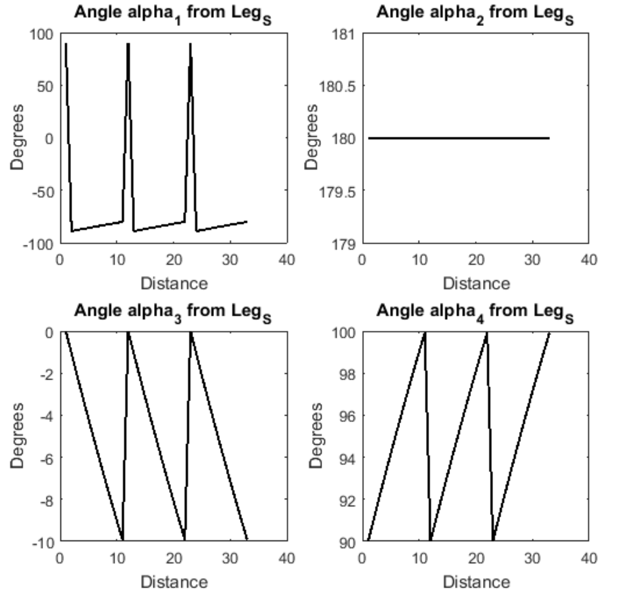

Fig. 6. Joints angles α_1, α_2, α_3 and α_4 from Leg_S in three walking cycles in sagittal plane.

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